

RELATIVE INVERSIONS ARE MAHONIAN AND TWO PROOFS OF MACDONALD SYMMETRY WHEN $q = 1$

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ABSTRACT. We introduce the notion of a relative inversion and show that they are Mahonian, i.e. that relative inversions are equidistributed with standard inversions on words. We then show that relative inversions can be used to describe inversion triples in Haglund's monomial description of (modified) Macdonald polynomials $\tilde{H}_\mu(X; q, t)$. We introduce a bijection between inversions and relative inversions, giving a direct combinatorial proof of the q, t -symmetry relation $\tilde{H}_\mu(X; q, t) = \tilde{H}_{\mu'}(X; t, q)$ at the specialization $q = 1$. Moreover, the bijection gives rise a family of new bijections on words that preserves the classical inv statistic.

Key words and phrases. Symmetric functions, Mahonian statistics, Macdonald polynomials, permutations, q -series, Young tableaux, bijections on words.